

Lösung 3-Graviton-Amplitude

2) Aus Aufgabenteil 1) haben wir

$$\begin{aligned}
 A_{ggg} &= \langle V_g(p_1) V_g(p_2) V_g(p_3) \rangle = \langle V_g\left(\frac{p_1}{2}\right) V_g\left(\frac{p_2}{2}\right) V_g\left(\frac{p_3}{2}\right) \rangle_L \langle V_g\left(\frac{p_1}{2}\right) V_g\left(\frac{p_2}{2}\right) V_g\left(\frac{p_3}{2}\right) \rangle_R \\
 &= (A_{ggg})_L^{tree} (A_{ggg})_R^{tree} \\
 &\sim \epsilon_{\mu\nu\lambda}^{\mu_1 \mu_2 \mu_3} t_{\mu\nu\lambda}^S(p_1, p_2, p_3) \epsilon_{\mu_1 \mu_2 \mu_3}^{\nu_1 \nu_2 \nu_3} t_{\nu_1 \nu_2 \nu_3}^S(p_1, p_2, p_3) \\
 &\stackrel{\epsilon_{\mu\nu\lambda}^{\mu_1 \mu_2 \mu_3} \rightarrow \mathcal{O}(\alpha^{10})}{=} \sum_1^{\mu\nu} \sum_2^{\nu\mu'} \sum_3^{\mu\mu'} (R_{2\mu} \eta_{\nu\mu'} + R_{3\nu} \eta_{\mu\mu'} + R_{1\mu} \eta_{\mu\nu'}) \\
 &\qquad\qquad\qquad (R_{2\mu} \eta_{\nu\mu'} + R_{3\nu} \eta_{\mu\mu'} + R_{1\mu} \eta_{\mu\nu'}) \\
 &= \sum_1^{\mu\nu} \sum_2^{\nu\mu'} \sum_3^{\mu\mu'} \\
 &\quad \left(\underline{R_{2\mu} \eta_{\nu\mu'} R_{2\mu} \eta_{\nu\mu'}} + \underline{R_{3\nu} \eta_{\mu\mu'} R_{2\mu} \eta_{\nu\mu'}} + \underline{R_{1\mu} \eta_{\mu\nu'} R_{2\mu} \eta_{\nu\mu'}} \right. \\
 &\quad + \underline{R_{2\mu} \eta_{\nu\mu'} R_{3\nu} \eta_{\mu\mu'}} + \underline{R_{3\nu} \eta_{\mu\mu'} R_{3\nu} \eta_{\mu\mu'}} + \underline{R_{1\mu} \eta_{\mu\nu'} R_{3\nu} \eta_{\mu\mu'}} \\
 &\quad \left. + \underline{R_{2\mu} \eta_{\nu\mu'} R_{1\mu} \eta_{\mu\nu'}} + \underline{R_{3\nu} \eta_{\mu\mu'} R_{1\mu} \eta_{\mu\nu'}} + \underline{R_{1\mu} \eta_{\mu\nu'} R_{1\mu} \eta_{\mu\nu'}} \right) \\
 &= \underline{\sum_1^{\mu\nu} R_{2\mu} R_{2\mu'} \sum_2^{\nu\mu'} \sum_3^{\mu\mu'} \eta_{\nu\mu'} \eta_{\mu\nu'}} \\
 &\qquad\qquad\qquad \sum_2^{\nu\mu'} \sum_3^{\mu\mu'} \\
 &\quad + \underline{\sum_2^{\nu\mu'} R_{3\nu} R_{3\nu'} \sum_1^{\mu\mu'} \sum_3^{\mu\mu'} \eta_{\nu\mu'} \eta_{\mu\nu'}} \\
 &\quad + \underline{\sum_1^{\mu\nu} R_{2\mu} \left(\sum_2^{\nu\mu'} R_{3\nu} \sum_3^{\mu\mu'} + \sum_3^{\mu\mu'} R_{1\mu} \sum_2^{\nu\mu'} \right)} \\
 &\quad + \underline{\sum_2^{\nu\mu'} R_{3\nu} \left(\sum_1^{\mu\mu'} R_{2\mu} \sum_3^{\mu\mu'} + \sum_3^{\mu\mu'} R_{1\mu} \sum_2^{\nu\mu'} \right)} \\
 &\quad + \underline{\sum_3^{\mu\mu'} R_{1\mu} \left(\sum_1^{\mu\nu} R_{2\mu} \sum_2^{\nu\mu'} + \sum_2^{\nu\mu'} R_{3\nu} \sum_1^{\mu\mu'} \right)}
 \end{aligned}$$

3) Das Ergebnis stimmt mit Gleichung (6) auf dem Übungsblatt überein. Für höhere α' -Ordnungen kommen Terme der folgenden Form hinzu:

$$\mathcal{O}(\alpha') \sim \sum_1^{\mu\nu} \sum_2^{\nu\mu'} \sum_3^{\mu\mu'} \left[\alpha' R_{2\mu} R_{3\nu} R_{1\mu} (R_{2\mu} \eta_{\nu\mu'} + R_{3\nu} \eta_{\mu\mu'} + R_{1\mu} \eta_{\mu\nu'}) + (\mu \leftrightarrow \mu', \nu \leftrightarrow \nu', \mu \leftrightarrow \mu') \right]$$

$$\sim \sum_1^{\mu\nu} R^4$$

$$\mathcal{O}(\alpha'^2) \sim \sum_1^{\mu\nu} \sum_2^{\nu\mu'} \sum_3^{\mu\mu'} 4\alpha'^2 R_{2\mu} R_{3\nu} R_{1\mu} R_{2\mu'} R_{3\nu'} R_{1\mu'} \sim \sum_1^{\mu\nu} R^6$$